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Abstract

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Keywords

reliability, softening, analysis, slopes, strain

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Reliability Analysis of Strain-Softening Slopes

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ABSTRACT: Slope reliability must be considered in the context of a variety of factors which control or influence stability or the balance between disturbing and resisting forces. The problems of slope stability are varied, ranging from first-time failures to reactivated landslides and from minor, localized slope movements to catastrophic events. Reliability analysis within a probabilistic framework offers a very powerful tool for taking into consideration the variability of key geotechnical parameters as well as other uncertainties. In this paper, the progressive decrease in shear strength along potential slip surfaces is considered in terms of a residual factor treated as one among several random variables in the slope reliability formulation. Attention is restricted to natural slopes in which the use of an 'infinite slope' model is appropriate and in which pore water pressure as well as the residual factor are very important. However, the proposed reliability approach can be extended to complex slopes with curved slip surfaces of arbitrary shape.

1 INTRODUCTION

1.1 *Residual Factor and Slope Stability*

The concept of peak and residual shear strength values is widely recognised in geomechanics and especially in relation to problems of slope stability. It is a consequence of the strain-softening mechanical behaviour of soil and rock.

Considering slope stability, peak shear strength is usually considered to be applicable to the analysis concerning potential first-time slope failures. On the other hand, residual shear strength is usually considered to be applicable to the analysis concerning the potential reactivation of existing landslides or slopes which have failed in the past. However, in many instances processes of progressive failure are important and it is often necessary or useful to consider the role of strain-softening in slope stability assessments either on a deterministic or probabilistic basis. Even when a slope has not suffered a complete failure or a landslide has not occurred, natural processes may lead to strain-softening within slopes. It is important to identify important situations of this type and the factors which are important.

1.2 *Definition of Residual Factor at a given location*

The significance of the difference between peak and residual values of shear strength increases with increasing brittleness of soil or rock. The term 'residual

factor' is related to the concept of residual shear strength and, at a given location, it is defined as the ratio:

$$R = \frac{\text{peak strength} - \text{current strength}}{\text{peak strength} - \text{residual strength}}$$

In general R may have any value between 0 and 1. However, if soil behaviour can be regarded as perfectly brittle, the shear strength will be either the peak value or the residual value. Any value in between peak and residual will not be feasible for a perfectly brittle material.

1.3 *Average Residual Factor for a slip surface in a slope*

The definition given above is appropriate as a local residual factor because it refers to a specific point or location along a surface within a slope. However the concept may also be applied to the whole of a surface within a slope. Such a surface may be a potential slip surface, planar or curved in shape.

The average shear strength along a potential slip surface may be at its peak value or at its residual value or at a value in between. Thus it is useful to consider the residual factor as the proportion of average shear strength decrease from a peak value to a residual value. The above definition still applies except that all the shear strength values are average values for the whole of the slip surface. Thus $R=0$ implies that the average shear strength along the whole of the slip surface has the peak value and $R=1$

implies that the average shear strength has the residual value.

A value of the average shear strength between the peak and the residual would imply a value of the average residual factor R above 0 and below 1.

Considering a slope in perfectly brittle soil, shear strength along a potential slip surface will be either at the peak or the residual. An alternative definition of average residual factor for a slip surface in a slope has been proposed as the ratio of length of slip surface at the residual shear strength, L_r , to the total length of the slip surface, L , i.e., $R = L_r / L$.

A value of $R=0$ implies that no strain-softening has occurred along any part of a potential slip surface. The shear strength at each point is to be calculated using the effective normal stress at that point and the peak shear strength parameters. At the other extreme, $R=1$ implies that strain-softening has occurred all along a potential slip surface. The shear strength at each point is, therefore, to be calculated on the basis of the effective normal stress at that point and the residual shear strength parameters.

1.4 Average Residual Factor between 0 and 1

This is the general case and there are at least three important reasons why it is necessary to consider a residual factor between 0 and 1.

(i) Firstly, there are slopes which have not suffered landsliding or observable instability but within which internal, localized shear movements or deformations may have occurred.

One reason for such internal deformations at any location within a slope could be the increase of pore water pressures which lead to decrease in local shear strength. If local shear stress exceeds the decreased shear strength in a strain-softening soil, shear strength will reduce below its peak value. Such local oversteering could lead to internal shear strains and deformations. For instance, a high rainfall event may not be of sufficient intensity and duration to cause landsliding at a given location and yet the pore pressure increases might be high enough to cause local oversteering, local failure and strain-softening.

Another reason for significant internal deformations could be the cumulative irreversible deformations due to earthquake shaking. For example, the magnitude of an earthquake may not be sufficiently high and its location with regard to an area not favourable to cause landsliding at that location. Yet, the cumulative irreversible deformations may be high enough to cause internal shear deformations and strain-softening.

The history of a region and, in particular, the frequency of external events such as high magnitude rainfall and earthquake events would be important in this regard.

(ii) Secondly, consider a slope which has failed in the past and of which future stability is to be as-

sessed. A potential slip surface may be located partly through failed areas and partly through areas which have not suffered significant deformations in the past. The residual factor of the first part would be 1 while of the second part would be equal or close 0. Thus the average value of the residual factor R for the whole slip surface would be between 0 and 1.

(iii) Thirdly, it is always important to assess the likelihood of progressive failure in slopes of soil or rock. If the slope materials are sufficiently strain-softening or brittle, local strain-softening and stress redistribution would occur during slope formation. Thus the residual factor will increase from an initial value of zero, associated with peak shear strength, to a higher value associated with average shear strength having fallen below the peak value. The factors which will determine the extent of progressive failure and thus the final value of the residual factor include the slope geometry, the history of slope formation, and the brittleness of the soil, the pore water pressures and any external disturbing agent in addition to the gravitational forces.

1.5 Basic expressions in terms of average, peak and residual shear strengths

From the original definitions considered earlier, the residual factor could be expressed as follows

$$R = \frac{s_p - s_{av}}{s_p - s_r} \quad (1)$$

in which s_p and s_r are the peak and the residual strengths respectively; and s_{av} is the average shear strength along a slip surface.

Consequently, the average shear strength may be expressed in terms of the residual factor as follows

$$s_{av} = R s_r + (1 - R) s_p \quad (1a)$$

1.6 Residual Factor in Slope Analysis

It is proposed that, in slope reliability studies, the residual factor be modelled as random variable along with other important variables. However, it is important to note that residual factor has been included as a variable in many studies over the last few decades. Following is a summary of the different options.

1.6.1 As a variable in deterministic studies

Within the framework of both limit equilibrium and stress-deformation analysis, many studies have, either directly or indirectly, considered the residual factor as a deterministic variable. The reader may refer, among many others, to Lo & Lee (1973) and Christian & Whitman (1969). Several examples and references are cited in a recent book (Chowdhury et al., 2010).

1.6.2 As a variable in probabilistic studies

Within the framework of probabilistic studies, the residual factor has also been used, directly or indirectly, as a deterministic variable although, in several studies, both the peak and residual shear strength parameters have been considered as random variables. The reader may refer, among others, to the papers by Chowdhury (1984, 1992), Chowdhury & A-Grivas (1982), Chowdhury et al. (1987), Tang et al. (1985) and, Chowdhury & Zhang (1993). Most of these references are concerned with exploration of the probability of progressive failure. Overviews of different models have been provided in a recent book Chowdhury et al. (2010). It is interesting that one of these models enables estimation of the most probable failure length along a potential slip surface in brittle strain-softening soil (Chowdhury et al. 1987, Tang et al., 1985). Consequently, the most probable value of the residual factor could be calculated for that potential slip surface. However, it must be emphasized that the residual factor was not considered directly as a variable in the model.

1.6.3 As a random variable

So far the residual factor has not been considered as a random variable in probabilistic slope analysis. In general, there are significant uncertainties concerning the magnitude of the residual factor and its spatial variability within a slope. Therefore, consideration of residual factor as a random variable seems justified in slope reliability studies within a probabilistic framework.

2 FORMULATION

2.1 Residual Factor and the 'Infinite Slope' model

Consider the well known 'infinite slope' model for the stability of a slope with a potential slip surface parallel to the ground surface assuming first that no strain-softening has occurred. Denote ground surface inclination by i , the vertical depth to potential slip surface by z , the unit weight of the soil by γ , the shear strength parameters by c' and $\tan \phi'$ and the dimensionless pore water pressure ratio by r_u . The factor of safety may be written as the ratio of average shear strength to average shear stress in the following simple form:

$$F = \frac{c' + \gamma z (\cos^2 i - r_u) \tan \phi'}{\gamma z \sin i \cos i} \quad (2)$$

Consider now a slope in which strain-softening has occurred and the residual factor is R . Let us differentiate between peak, residual and average shear strength by using the appropriate suffixes for the shear strength parameters.

From Eq 1(a) and Eq 2, it is now easy to show that the factor of safety, for a slope in which strain-softening has occurred over part of the slip surface, is given by the following expression

$$F = \frac{R\{c'_r + \gamma z (\cos^2 i - r_u) \tan \phi'_r\} + (1 - R)\{c'_p + \gamma z (\cos^2 i - r_u) \tan \phi'_p\}}{\gamma z \sin i \cos i} \quad (3)$$

For slope stability, the shear strength parameters, the pore water pressure ratio, and the residual factor can all be considered as important parameters. Thus there can be up to 6 random variables [$c'_p, c'_r, \phi'_p, \phi'_r, r_u$ and R] as part of a reliability analysis.

2.2 Reliability Index and Probability of Failure

2.2.1 Reliability Index

A widely accepted and simple definition of reliability index is the ratio of expected safety margin to the standard deviation of the performance function which, in this case, is the factor of safety F . The corresponding expression in terms of the expected value of factor of safety, $E(F)$ or \bar{F} , and the standard deviation of F , σ_F , is

$$\beta = \frac{E(F) - 1}{\sigma_F} = \frac{\bar{F} - 1}{\sigma_F} \quad (4)$$

Here the factor of safety is a function of six important parameters including the residual factor and the pore water pressure; all or some of these parameters may be regarded as random variables.

Several numerical methods have been developed for estimating the statistical moments of a performance function dependent on multiple random variables. For practical purposes the two important statistical moments to be estimated are the expected value and the variance (square of standard deviation) and these methods have been reviewed recently (Chowdhury et al., 2010).

2.2.2 Probability of Failure

Having calculated the reliability index an assumption must be made about the probability distribution for the factor of safety. Considering the performance function F to follow either a normal or a lognormal distribution, the probability of failure p_F can be calculated using one of a number of available methods as summarized by Chowdhury et al. (2010).

2.3 Simplified expressions for single random variables

2.3.1 Pore pressure ratio r_u as the only random variable

For this special case, one can rewrite Eq. 3 as follows.

$$F = A - B r_u \quad (5)$$

where,

$$A = \frac{R c'_r + (1-R)c'_p + \gamma z \cos^2 i \{ \tan \phi'_p - R(\tan \phi'_p - \tan \phi'_r) \}}{\gamma z \sin i \cos i}$$

$$\text{and, } B = \frac{\tan \phi'_p - R(\tan \phi'_p - \tan \phi'_r)}{\sin i \cos i}$$

From Eq. 5 (denoting the mean of r_u by \bar{r}_u),

$$\beta = \frac{[A - B \bar{r}_u] - 1}{B \sigma_{r_u}} \quad (6)$$

2.3.2 Residual factor R as the only random variable

Again one may rearrange the expression for F (Eq.3) by combining all terms containing R. Thus one can write:

$$F = M - NR \quad (7)$$

in which

$$M = \frac{c'_p + \gamma z (\cos^2 i - r_u) \tan \phi'_p}{\gamma z \sin i \cos i}$$

$$N = \frac{(c'_p - c'_r) + (\cos^2 i - r_u) \gamma z (\tan \phi'_p - \tan \phi'_r)}{\gamma z \sin i \cos i}$$

The reliability index is

$$\beta = \frac{(M - N\bar{R}) - 1}{N \sigma_R} \quad (8)$$

2.3.3 Natural slopes with $c' = 0$ [$c'_r = 0, c'_p = 0$]

For these cases much more simplified forms for the factor of safety can be obtained by substituting $c'_r = 0$ and $c'_p = 0$ in Eq. 3. Often only $c'_r = 0$ and $c'_p \neq 0$, for which also Eq. 3 may be appropriately simplified. Consequently, expression for the constants A and B, in Eq 5, and that for M and N in Eq 7 will also be revised.

3 ILLUSTRATIVE EXAMPLE

The above formulation for the analysis of strain-softening slopes is illustrated below with the help of an example problem of a natural slope which can be analysed on the basis of the infinite slope model.

3.1 Slope Description and Assumed Data

A homogeneous infinite slope in cohesive soil is considered in which seepage is occurring parallel to the slope. The top flow line may be located at any depth below the surface of the slope and above the

potential slip surface. Thus the pore water pressure ratio may have any value between 0 and approximately 0.5.

In this example, the following data are assumed: The slope has an inclination $i = 12^\circ$, depth to potential failure surface $z = 3\text{m}$, bulk unit weight of soil $\gamma = 20 \text{ kN/m}^3$. The mean values of the shear strength parameters are: $c'_p = 5.0 \text{ kPa}$, $c'_r = 0.0$, $\tan \phi'_p = 0.36$ and $\tan \phi'_r = 0.26$. The values of coefficient of variation (c.o.v.) of different random variables are assumed in section 3.3.

3.2 Deterministic Analysis

The factor of safety is the index of stability status of the slope and its value can be obtained from either Equation (3) when the mean values of the pore water pressure ratio r_u , the shear strength parameters (peak and residual), and the residual factor, R are considered. Let us first consider one calculation for the case when the mean r_u is assumed to be 0.2 and, for the strain-softening situation ($0 < R < 1$), the mean residual factor is assumed to be $R = 0.2$. For this case, the mean factor of safety is found to be $F = 1.59$. For the sake of comparison, in the two extreme cases when the entire length of the potential slip surface is at the peak strength ($R=0$), or at the residual strength ($R=1$), the corresponding values of F are obtained as 1.75 and 0.97 respectively. Thus there is a significantly large range of values of calculated F considering the possible range of values of the residual factor R.

In order to study the variation of the factor of safety F with the variation of the pore water pressure and the residual factor, values of F have been obtained for a range of values of r_u (between 0.0 and 0.5), and for a range of values of R (between 0.0 and 1.0), while the four shear strength parameters have been kept at their mean values. The results are presented in Table 1. We note that, for any given value of the pore pressure ratio, the factor of safety is strongly dependent on the value of the residual factor R. Assuming the entire length of the potential slip surface to be at the peak strength ($R=0.0$) could lead to a significant overestimation of stability on the unsafe side. Conversely, assuming the entire length of the potential slip surface to be at the residual strength ($R=1.0$) could lead to a significant underestimation of the stability.

Table 1: Values of F for different values of R and r_u

R	Values of Pore Pressure Ratio r_u					
	0.0	0.1	0.2	0.3	0.4	0.5
0.0	2.10	1.93	1.75	1.57	1.40	1.22
0.2	1.93	1.76	1.59	1.43	1.26	1.09
0.4	1.75	1.59	1.44	1.28	1.12	0.97
0.6	1.58	1.43	1.28	1.13	0.99	0.84
0.8	1.40	1.26	1.12	0.99	0.85	0.71
1.0	1.22	1.10	0.97	0.84	0.71	0.58

3.3 Reliability Analysis

In the reliability analysis of such a slope, as discussed earlier, the performance function is given by Eq 3 or Eq 3(a) which shows that, in the strain-softening situation, there can be up to six random variables [c'_p , c'_r , $\tan \phi'_p$, $\tan \phi'_r$, r_u and R]. It is assumed that all the six random variables are normally distributed and uncorrelated.

The reliability analysis is carried out using a simpler method, namely, the Mean-Value-First-Order Second-Moment method (MVFOSM). As in the deterministic analysis, mean values of r_u and R are both taken as 0.2. Assuming a coefficient of variation (c.o.v.) of 0.2 for all the random variables, the reliability index β is obtained as 2.48. Assuming further that the factor of safety is normally distributed, the probability of failure $p_F = \Phi(-\beta)$ is obtained as 6.57×10^{-3} . Thus it is seen that even when a deterministic analysis indicates a safe slope ($F = 1.59$), a probabilistic analysis with a likely value of c.o.v. of 0.2 indicates a substantial failure probability.

For high pore water pressure situation, the results are more striking: when mean $r_u = 0.4$, the results indicate a safe slope again with $F = 1.26$ (Table 1); and yet $\beta = 1.17$, and $p_F = 1.2 \times 10^{-1}$ which is hardly acceptable. The necessity for and importance of a reliability analysis under probabilistic framework is thus quite evident. Tables 2 and 3 present the detailed results: Table 2 presents the variation of the reliability index β when the mean value of R is varied from 0.2 to 0.8, and that of r_u is varied from 0.0 to 0.5. Table 3 presents the corresponding probabilities of failure p_F . The results presented above are those obtained when all the six important parameters are treated as random variables.

In order to bring out how the reliability index or the probability of failure varies with the uncertainties in the important parameters, computations are also carried out considering (i) r_u as the only random

Table 2: Values of β for different mean values of R and r_u

R	Values of Pore Pressure Ratio r_u					
	0.0	0.1	0.2	0.3	0.4	0.5
0.2	3.25	2.93	2.48	1.89	1.17	0.41
0.4	3.11	2.69	2.13	1.42	0.62	-0.174
0.6	2.52	2.04	1.43	0.705	-0.079	-0.836
0.8	1.59	1.14	0.58	-0.07	-0.77	-1.46

Table 3: Values of p_F for different mean values of R and r_u

r_u	Values of Residual Factor R			
	0.2	0.4	0.6	0.8
0.0	5.746×10^{-4}	9.26×10^{-4}	5.87×10^{-3}	5.58×10^{-2}
0.1	1.696×10^{-3}	3.53×10^{-3}	2.06×10^{-2}	1.282×10^{-1}
0.2	6.57×10^{-3}	1.67×10^{-2}	7.61×10^{-2}	2.81×10^{-1}
0.3	2.96×10^{-2}	7.79×10^{-2}	2.40×10^{-1}	5.27×10^{-1}
0.4	1.203×10^{-1}	2.67×10^{-1}	5.31×10^{-1}	7.79×10^{-1}
0.5	3.41×10^{-1}	5.69×10^{-1}	7.98×10^{-1}	9.34×10^{-1}

variable, (ii) R as the only random variable, (iii) both r_u and R are random variables, (iv) five random variables excluding R , and (v) all six parameters as random variables, as described below. In order to bring out the differences clearly, the mean values of r_u and R are taken as 0.3 and 0.4 respectively, and a coefficient of variation (c.o.v.) of 0.2 has been assumed for all the random variables. The results are presented in Table 4.

Table 4: Comparison of different cases (for data- see above)

Cases	F	β	p_F
r_u as the only random variable	1.28	2.96	1.55×10^{-3}
R as the only random variable	1.28	4.76	9.47×10^{-7}
r_u and R as random variables	1.28	2.51	5.98×10^{-3}
5 random variables excluding R	1.28	1.49	6.86×10^{-2}
6 random variables including R	1.28	1.42	7.80×10^{-2}

From Table 4 it can be observed that reliability within a probabilistic framework adds significantly to the information concerning slope reliability. Deterministic analysis shows a single value of the factor of safety. In contrast, we note that the reliability index decreases and the probability of failure increases as the number of random variables is increased from one to six.

In this particular example, the inclusion as a random variable of the pore pressure ratio r_u has the most dominating influence on β and p_F . The inclusion as a random variable of the residual factor R has relatively less significant influence on β and p_F . With a higher assumed value of the coefficient of variation of R , its influence on the reliability index and probability of failure would indeed be more significant.

Further work must be carried out to study the influence of variation in the statistical parameters of the residual factor R . In particular, for a given mean value of R , its standard deviation must be varied to study the influence on estimated values of reliability index and probability of failure.

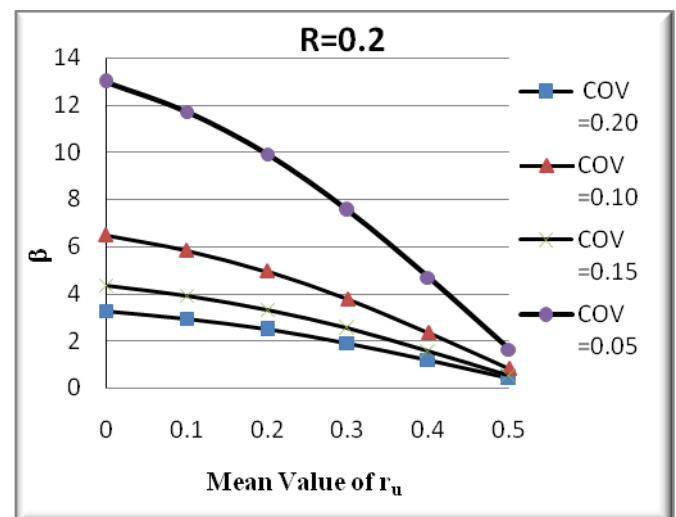


Figure 1: Plot of β versus mean r_u for $R=0.2$

Moreover, it is important to note that there may be a significant uncertainty associated with the mean value of the residual factor R . With current knowledge, it is not possible to estimate the mean value of the residual factor except perhaps for specific cases of very well-documented slope failures. In other words, while a probability distribution may be assumed for the residual factor R (implying an interpreted mean and a standard deviation), the mean value of R itself may be regarded as a random variable. An important aspect of future research should deal with the interpretation of long-term data on slope performance so that insight can be gained into the residual factor as a random variable.

For the illustrative example considered in this paper, it is of interest to study the effect of variation of the statistical parameters of r_u on the reliability index and the probability of failure (considering all six parameters as random variables with equal values of c.o.v.). Assuming specific values of the mean and the c.o.v. of the residual factor R , both the mean and c.o.v. of the pore pressure parameter have been varied and the corresponding estimated values of β plotted against mean values of r_u for various values of coefficient of variation (c.o.v.) as in Fig. 1.

The values of the probability of failure corresponding to values of β are also obtained. The variation of p_F with mean values of r_u are plotted in Fig. 2.

From these plots it is observed that (i) as the pore water pressure becomes large with mean value of r_u approaching 0.5, the coefficient of variation (c.o.v.) becomes almost irrelevant as all the curves (except the one with a very small c.o.v. of 0.05) tend to converge to a minimum value of β and a maximum value of p_F . (ii) Again, when r_u is low to medium (say, between 0 to 0.3), the effect of c.o.v. on β and p_F is rather pronounced.

4 CONCLUDING REMARKS

Internal deformations and strain-softening within slopes can have significant effects on their stability and reliability. The processes which lead to dynamic changes in slope stability and performance over time have been recognized for several decades. However, the associated implications for the interpretation of slope reliability have not been fully understood. In this paper strain-softening effects along a potential slip surface within a natural slope are included in terms of a parameter already known to geotechnical engineers as a 'residual factor'. It is shown that such a parameter may be included both in deterministic and probabilistic studies. For a natural slope, the relevant equations are presented as a modification of the well known 'infinite slope' model.

Considering the whole of a potential slip surface to be either at the peak strength or at the residual strength can lead to a significant overestimate or a

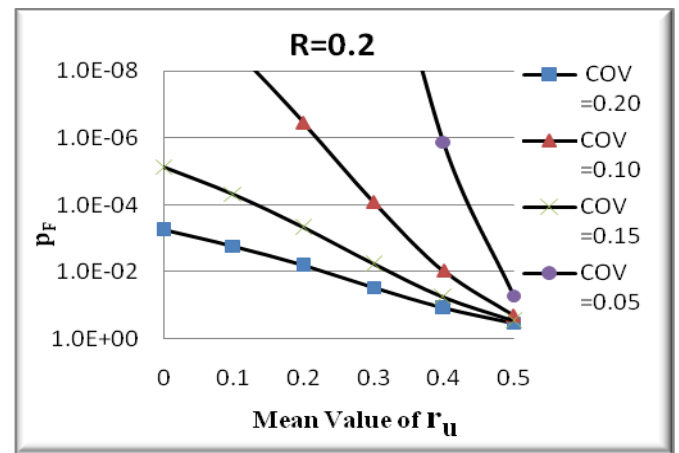


Figure 2: Plot of p_F versus mean r_u for $R=0.2$

significant underestimate of slope reliability. Therefore, it is important to develop methods which include the residual factor as an important variable.

An illustrative example has been included in this paper and some parametric studies are presented in which the variation of both the reliability index and the probability of failure are studied. For the particular set of data assumed in the illustrative example the relative significance of the choice of the number of random variables is discussed.

Further research is necessary to develop methods for assessing the probability distribution of the residual factor from long-term field data. Such data should include case studies of long-term slope monitoring as well as back-analyses of well documented landslides.

REFERENCES

- Chowdhury, R. 1984. Recent Developments in Landslide Studies-Probabilistic Methods, State of the Art Report, Proc. *International Symposium on Landslides*, Toronto, Vol. 1, 209-229
- Chowdhury R. 1992. Simulation of Risk of Progressive Slope Failure, *Canadian Geotechnical Journal*, Vol. 29, 1, 94-102.
- Chowdhury R. and A-Grivas, D. 1982. Probabilistic Model Of Progressive Failure Of Slopes, *Journal of Geotechnical Engineering Division*, ASCE, 108, GT6, 803-819.
- Chowdhury, R. and Zhang, S., 1993. Modelling the Risk of Progressive Slope Failure: A New Approach, *Reliability Engineering and System Safety*, 40, 17-30.
- Chowdhury, R. Flentje, P. and Bhattacharya, G., 2010. *Geotechnical Slope Analysis*, CRC Press/ Balkema, 737pp.
- Chowdhury, R. Tang, W.H. and Sidi, I., 1987. Reliability Model of Progressive Slope Failure, *Geotechnique*, 37, 467-481
- Christian, J.T. and Whitman, R.V., 1969. A one-dimensional model for progressive failure, *Proc. 7th Int. Conf. Soil Mech. Found. Eng.*, 2; 541-545.
- Lo, K.Y. and Lee, C.F., 1973. Stress analysis and slope stability in strain softening materials, *Geotechnique*, 23, 1; 1-11.
- Tang, W.H., Chowdhury R. and Sidi, I., 1985. Progressive Failure Probability of Soil Slopes, ICOSAR'85, 4th Int. conf on Structural Safety and Reliability, Vol. III, III 363-III 372.